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MISSOURI UNIV-COLUMBIA DEPT OF ELECTRICAL ENGINEERING  
ANALYSIS OF INHERENT ERRORS IN ASYNCHRONOUS DIGITAL FLIGHT CONT--ETC(U)  
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## 1.0 INTRODUCTION

Current research on flight control systems for military aircraft emphasizes digital control. Redundant digital hardware is frequently needed to achieve the required safety-of-flight reliability, and certain additional design considerations and performance limitations are associated with the use of redundant hardware.

When the redundant hardware is operated in an asynchronous manner, errors arise that are attributed solely to asynchronism. These errors are present even when the hardware is functioning in a normal manner. The errors are small, but not negligible and they influence dynamic performance; also, quantitative bounds for such "inherent errors" are needed for the design of voting algorithms and to permit the distinction between normal operation and operation in the presence of equipment failure.

This report summarizes recent progress achieved under an Air Force grant for the study of inherent errors in asynchronous digital flight controls. The research involves three areas:

1. Model Development
2. Software Development
3. Parametric Analysis of Representative Flight Control Systems

The results for each of these areas are summarized below. A fuller description of these results will appear in an AFOSR Scientific Report, which is in preparation.

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## 2.0 MODEL DEVELOPMENT

Three models have been developed for the closed-loop dynamic operation of an aircraft with dual-redundant asynchronous digital controllers. All three models have the following characteristics<sup>1</sup>: (1) Asynchronous sampling is modeled by skewed, synchronous sampling in which each redundant digital controller operates at exactly the same rate but there is a fixed time skew for each one. (2) Although there are two digital controllers only one is used to obtain the signals fed back to control the aircraft; the other controller does not influence the aircraft performance but it is used for computing inherent errors. (3) The aircraft, sensors, and actuators are modeled by linear, continuous-time state-variable equations.

### 2.1 Model I

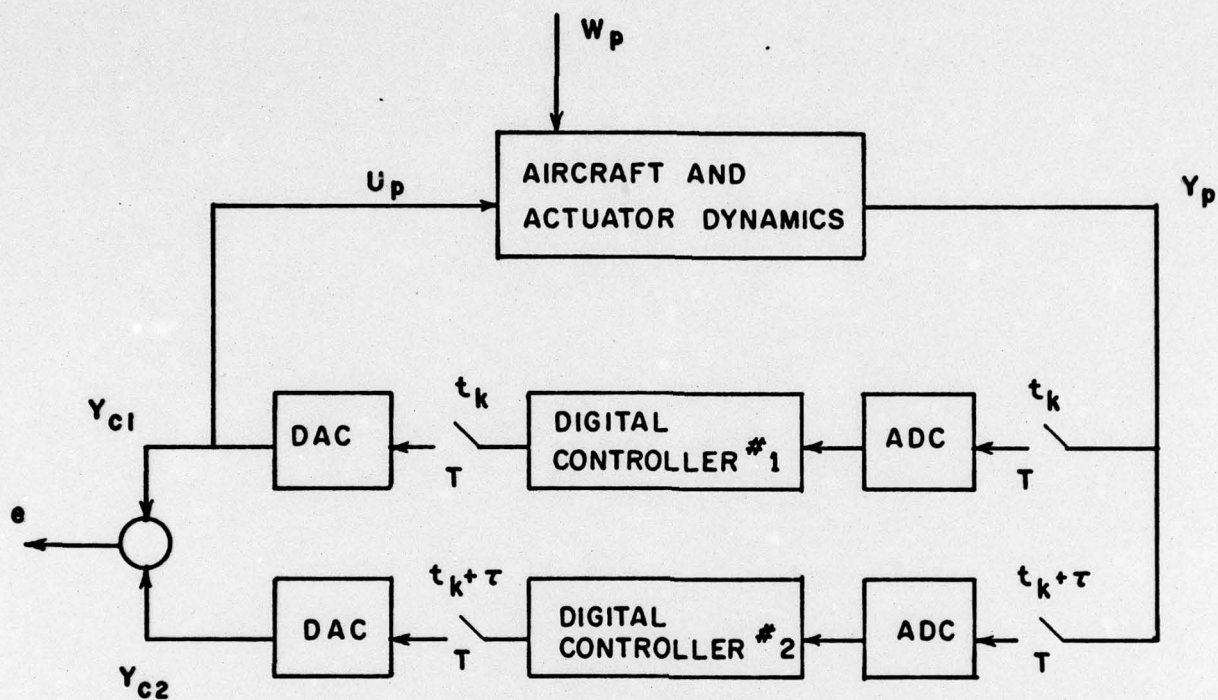
Figure 1 shows the block diagram of the first model. This model has the following set of first-order discrete-time equations

$$x(t_{k+1}) = F(T, \tau) x(t_k) + G(t_k, t_{k+1}, \tau, w_p(t))$$

where  $x(t_k)$  is a combined state vector consisting of the plant variables  $x_p$  and the digital controller variables  $x_{c1}$  and  $x_{c2}$ , as

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{c1}(t_k) \\ x_{c2}(t_k) \end{bmatrix}$$

$F(t, \tau)$  is a matrix that depends on  $T, \tau$ , and the aircraft-model parameters, and  $G(t_k, t_{k+1}, \tau, w_p(t))$  is a vector that depends on  $w_p(t)$  as well.



ADC: ANALOG-TO-DIGITAL CONVERTER  
 DAC: DIGITAL-TO-ANALOG CONVERTER  
 T: SAMPLE PERIOD  
 $\tau$ : SKEW

FIGURE 1 BLOCK DIAGRAM FOR MODEL I



The inherent error for the first model is shown as  $e(t)$  on Figure 1. This error is piecewise constant:

$$e(t) = \begin{cases} e_A(t) = y_{c1}(t_k) - y_{c2}(t_k + \tau) & \text{for } t_k + \tau \leq t < t_{k+1} \\ e_B(t) = y_{c1}(t_{k+1}) - y_{c2}(t_k + \tau) & \text{for } t_{k+1} \leq t < t_{k+1} + \tau \end{cases}$$

The inherent error can be characterized in a statistical manner. Assume that the external input  $w_p$  is a sample function from a Gaussian white noise random process with zero mean. (In a typical flight-control application this input could be used to simulate the spectrum of frequencies of the actual pilot input by first passing the white noise signal through an appropriate linear filter; the filter is chosen so that the power spectral density of its output is representative of that of a typical pilot input; the dynamic equations of the filter are incorporated into the aircraft-model state equations.) The covariance matrix of the states is defined as

$$P_x(k) = E[x(t_k) x^T(t_k)]$$

where  $E$  is the expected-value operator.  $P_x(k)$  is used in the statistical characterization of the inherent errors.

The steady-state covariance of the states,  $P_{xss}$ , is found by solving the equation

$$P_{xss} = F(T, \tau) P_{xss} F^T(T, \tau) + V(T, \tau)$$

where  $V(T, \tau)$  is a matrix that can be calculated from system matrices and the covariance matrix for  $w_p$ . Once  $P_{xss}$  is obtained, the steady-state covariance

matrices of the inherent errors,  $e_A$  and  $e_B$ , can be obtained by matrix multiplication. The numerical values of the elements of  $P_{xss}$  give the variances of the inherent errors about their means of zero.

All required equations for the state-variable model and the statistical analysis have been developed and are given in References 1 and 2.

### 2.3 Model II

The second model incorporates the same assumptions as the first model but also allows for three time delays, associated with the delays due to data transmission and control-signal computations. Model II assumes sampling at a single rate and includes sampling of the pilot input.

The three time delays are:

$\delta_a$ : A/D conversion and transfer time

$\delta_c$ : digital-controller computation time

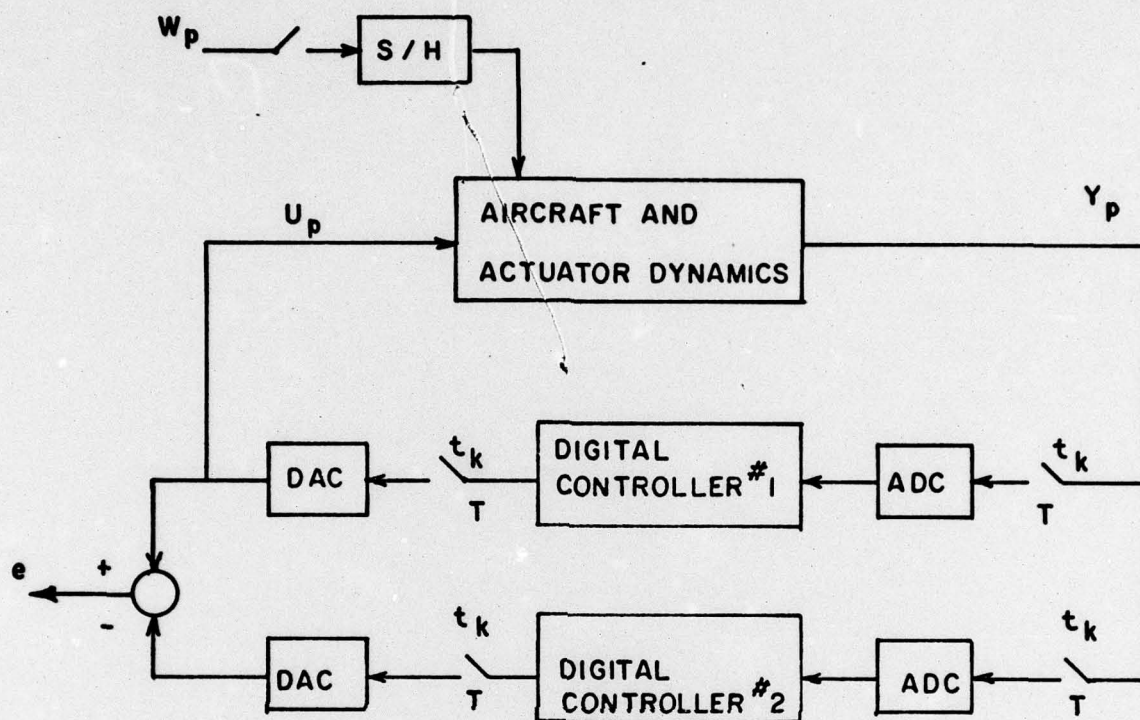
$\delta_d$ : D/A conversion and transfer time

Figure 2 shows the basic configuration assumed and Table 1 gives the event times where relevant changes in the system variables occur.

The state equations for Model II involve the new variables  $x_{hp1}$  and  $x_{hp2}$ , which are needed to provide the delay in the piecewise-constant samples of the  $x_p$ , for digital controllers 1 and 2, respectively. Thus the state vector for Model II is

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{hp1}(t_k) \\ x_{c1}(t_k) \\ x_{hp2}(t_k + \tau) \\ x_{c2}(t_k + \tau) \end{bmatrix}$$





ADC : ANALOG - TO - DIGITAL CONVERTER

DAC : DIGITAL - TO - ANALOG CONVERTER

S/H : SAMPLE - AND - HOLD

$T$  : SAMPLE PERIOD

$\tau$  : SKEW

FIGURE 2 BLOCK DIAGRAM FOR MODEL II



Table 1. Event Times for Model II

Time	Events
$t_k$	$y_p$ sampling initiated for digital controller 1. $w_p$ sampling initiated.
$t_k + \delta_a$	$y_p$ reaches digital controller 1.
$t_k + \delta_a + \delta_c$	$y_{c1}$ computation is complete.
$t_k + \delta_a + \delta_c + \delta_d$	$y_{c1}$ reaches the actuator.
$t_k + \tau$	$y_p$ sampling initiated for controller 2.
$t_k + \tau + \delta_a$	$y_p$ reaches controller 2.
$t_k + \tau + \delta_a + \delta_c$	$y_{c1}$ computation is complete.

and the state equations are

$$x(t_{k+1}) = F(T, \tau) x(t_k) + G(T, \tau) w_p(t_k)$$

The matrices  $F$  and  $G$  in this equation are different from those in the state equation for Model I; in particular,  $G(T, \tau)$  is dependent on  $T$  and  $\tau$  but not on  $w_p(t)$  since the pilot input is sampled in Model II.

The inherent error is again piecewise-constant and is defined as

$$e(t) = \begin{cases} e_A(t) = y_{c1}(t_k + \delta_a + \delta_c) - y_{c2}(t_k + \tau + \delta_a + \delta_c) & \text{for } t \text{ in Interval A} \\ e_B(t) = y_{c1}(t_{k+1} + \delta_a + \delta_c) - y_{c2}(t_k + \tau + \delta_a + \delta_c) & \text{for } t \text{ in Interval B} \end{cases}$$

where Interval A is  $[t_k + \tau + \delta_a + \delta_c, t_{k+1} + \delta_a + \delta_c)$  and Interval B is  $[t_{k+1} + \delta_a + \delta_c, t_{k+1} + \tau + \delta_a + \delta_c)$ ,  $k = 0, 1, 2, \dots$

The covariance analysis for Model II follows the same procedure that was used in Model I. The covariance matrix for the states is defined as

$$P_x(k) = E[x(t_k) x^T(t_k)]$$

and it satisfies the difference equation

$$P_x(k+1) = F P_x(k) F^T + G W G^T$$

The covariance matrices for  $e_A(t)$  and  $e_B(t)$  are related to  $P_x(k)$  as follows

$$P_{eA}(k) = (H_1 - H_2) P_x(k) (H_1 - H_2)^T + \rho W \rho^T$$

$$P_{eB}(k) = (H_1 F - H_2) P_x(k) (H_1 F - H_2)^T + (H_1 G - \rho) W (H_1 G - \rho)^T$$

where  $H_1$ ,  $H_2$ , and  $\rho$  are constant matrices.



### 2.3 Model III

The third model incorporates all the assumptions of the first model but also allows for multirate sampling; that is, the pilot input  $w_p$  is now sampled at a slower rate than the rate at which the control system operates. Figure 3 shows the configuration for Model III. Note that the digital controller sampling rates are  $n$  times the pilot-input sampling rate. ( $n$  is a positive integer.)

The state equations for Model III take the form

$$x(t_{k+1}) = F(T, \tau) x(t_k) + G(T, \tau) w_p(t_k)$$

where  $F$  and  $G$  are different from, but play the same role as  $F$  and  $G$  in the equations for Models I and II.

The inherent error is again piecewise-constant, and it changes  $2n$  times in the time interval from  $t_k + \tau$  to  $t_k + T$ :

$$e(t) = \begin{cases} e_{Ai}(t) = y_{c1}(t_k + \frac{i}{n}T) - y_{c2}(t_k + \frac{i}{n}T + \tau) & \text{for } t_k + \frac{i}{n}T \leq t < t_k + \frac{i+1}{n}T \\ e_{Bi}(t) = y_{c1}(t_k + \frac{i+1}{n}T) - y_{c2}(t_k + \frac{i}{n}T + \tau) & \text{for } t_k + \frac{i+1}{n}T \leq t < t_k + \frac{i+1}{n}T + \tau \end{cases}$$

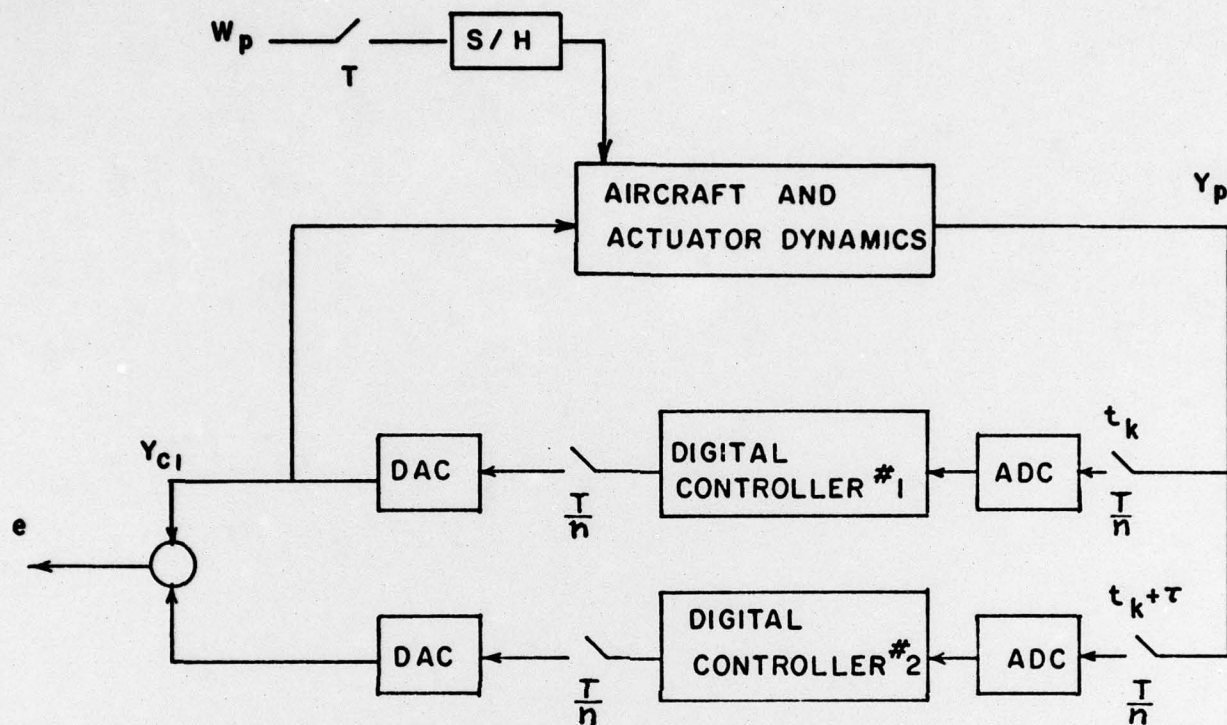
$$i = 0, 1, \dots, n-1, \quad k = 0, 1, 2, \dots$$

It is possible in this case to define "average errors"  $E_A$  and  $E_B$ , as

$$E_A = \frac{1}{n} \sum_{i=0}^{n-1} e_{Ai}$$

$$E_B = \frac{1}{n} \sum_{i=0}^{n-1} e_{Bi}$$





ADC : ANALOG -TO- DIGITAL CONVERTER

DAC : DIGITAL -TO- ANALOG CONVERTER

S/H : SAMPLE- AND- HOLD

$T$  : PILOT - INPUT SAMPLE PERIOD

$T/n$  : DIGITAL - CONTROLLER SAMPLE PERIOD

$\tau$  : SKEW

FIGURE 3 BLOCK DIAGRAM FOR MODEL III

and to obtain closed-form expressions for both, in terms of  $x(t_k)$  and  $w_p(t_k)$ .

The covariance analysis for Model III is complicated by the presence of multiple rates. Define the covariance matrix of the state variables by

$$P_x(k,m) = E[x(t_k + \frac{m}{n} T) x^T(t_k + \frac{m}{n} T)]$$

for

$$m = 0, 1, \dots, n-1, \quad k = 0, 1, 2, \dots$$

Then it can be shown that

$$P_x(k,m+1) = \Delta F P_x(k,m) \Delta F^T + \Delta G W_m \Delta G^T$$

where  $\Delta F$  and  $\Delta G$  are known functions of the system parameters and sample periods and  $W_m$  is the input covariance matrix. The above equation requires a starting value,  $P_x(k,0)$ , which may be obtained from

$$P_x(k+1,0) = F(T,\tau) P_x(k,0) F^T(T,\tau) + G(T,\tau) W_0 G^T(T,\tau)$$

These equations are also useful for studying the steady-state covariance matrices for the state variables.

The  $2n$  covariance matrices for the inherent errors are related to the covariance matrices for the states in a straightforward manner.



### 3.0 SOFTWARE DEVELOPMENT

The use of the digital computer is needed to use the models for dynamic simulations and covariance analyses of realistic flight-control systems. A few low-order, simplified closed-loop systems have been analyzed by hand with Model I,<sup>1,2</sup> but practical systems must be studied by using specially written software packages. There is a separate package for each model.

These packages are described briefly below. Details for the package for Model I are in References 1, and are in preparation for Models II and III.

#### 3.1 Model I Software

The software package for Model I consists of a FORTRAN main program and 16 subroutines.<sup>1,2</sup> There are approximately 1500 cards in the package. Most of the subroutines are taken from the software package DIGIKON, which was written under contract with the Air Force Flight Dynamics Laboratory by the Honeywell Flight Systems Division.<sup>3</sup>

The basic input to the software package is the state variable description of the aircraft dynamics and the digital controllers. Output consists of the description of the closed-loop, discrete-time state equations, the steady-state covariance matrices for the states and the inherent error, and the system response to a unit-step pilot input.

#### 3.2 Model II and Model III Software

There are distinct software packages for Model II and for Model III. They are similar in structure and function to the software for Model I and make use of the same basic DIGIKON subroutines.



#### 4.0 PARAMETRIC ANALYSES

The practical system chosen for study using the models and software previously described is the pitch axis flight-control system of the A-7D tactical fighter. This aircraft is the one whose dynamics are being simulated in the Flight Control Facility of the Air Force Flight Dynamics Laboratory's Digital Avionics Information System (DAIS) Advanced Development Program.<sup>4</sup> The purpose of the DAIS program is to demonstrate a coherent solution to the costly problem of proliferation and nonstandardization of aircraft avionics through modular digital hardware and software concepts. The program, managed by the Air Force Avionics Laboratory, is a joint effort of several Air Force organizations and includes the Air Force Flight Dynamics Laboratory.

References 1 and 2 contain discussions of the results of applying Model I to a version of the A-7D pitch axis flight-control system. The major system components include the pilot-input, the aircraft and actuator dynamics, and dual-redundant asynchronous digital controllers. The first two of these components comprise a 4th-order linear, continuous system and the controllers are each first-order.

The results of the study indicate that, under typical conditions of flight (nominal forward velocity in the cruise condition at sea level) the absolute value of the largest inherent error in the elevator-deflection command is 3.20 percent of the steady-state value. When a zero-mean, white-noise input is applied, the covariance analysis revealed a steady-state inherent comparison error of 3.13 percent maximum. See References 1 and 2 for a more complete discussion.

Work is progressing on applying all three models to a slightly modified version of the A-7D pitch axis control system. This version follows the equations programmed in analog-computer simulation that is currently being used in the DAIS program.<sup>5</sup> The linear dynamics are 6th order and the controller equations are again first-order. These studies will be reported in the AFOSR Scientific Report which was mentioned in Section 1.0 and which is in preparation.



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Progress has been achieved in the study of inherent errors in asynchronous digital flight controls. The research involves model development, software development, and parametric analysis of representative flight control systems.		